A Noninformative Prior Bayesian Approach to Reliability Growth Projection

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Outline.

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- Outline of Approach to Failure Rate Projection.
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- Reliability Projection using Parsimonious Model Approximation.
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- Noninformative Prior Bayesian Approach w/ A and B-Mode Classification.
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- Extensions of Approach where not all Fixes are Assumed Delayed.
- Concluding Remarks.

Problem and Assumptions.

Problem: Assess impact of delayed corrective actions (fixes) at completion of test phase.

Model Assumptions:

1. k potential failure modes where k is large.
2. Each failure mode has constant failure rate over test phase.
3. Occurrence of failures due to modes are statistically independent.
4. Each failure mode occurrence causes system failure.
5. Corrective actions are implemented at end of test phase.
6. At least one failure mode has a repeat failure.
7. Mode failure rates are a realization of a random sample of size k from a gamma distribution with density

$$f(x) = \begin{cases} \dfrac{x^{\alpha} e^{-x/\beta}}{\Gamma(\alpha+1)\beta^{\alpha+1}} & x > 0; \\ 0 & otherwise \end{cases}$$

$$where\ \alpha > -1, \beta > 0$$

Outline of Approach.

True system failure rate after corrective actions to observed failure modes in test denoted by $\rho(T)$, given by,

$$\rho(T) = \sum_{i \in obs} (1 - d_i)\lambda_i + \sum_{i \in unobs} \lambda_i$$

1. For each observed failure mode, assess fix effectiveness factor (FEF) d_i by d_i^*. Assessment based on analysis of failure mechanism(s) that give rise to $n_i > 0$ observed failures due to mode.
2. For each $i \in obs$ and $i \in unobs$, assess unknown mode failure rate $x_i \in \{\lambda_1, \ldots, \lambda_k\}$ based on observed data o_i.
   a) Using noninformative prior, $u(\lambda_i)=1/k$ for $i=1,\ldots,k$, obtain posterior density $g(x_i|o_i)$ for X_i.
   b) Obtain expected value of posterior, denoted by $E[X_i|o_i]$. Will be in terms of k and $\lambda_1,\ldots,\lambda_k$.
   c) Express $E[X_i|o_i]$ in terms of recognizable quantities that can be represented by a parsimonious model.
   d) Statistically estimate parameters of model.
   e) Express projected failure rate in terms of estimated model parameters based on assuming k potential failure modes.
   f) Find the limit of the finite k failure rate projection as $k \rightarrow \infty$.

Posterior Mean.

Observed mode $i \in obs$ has unknown failure rate $x_i \in \{\lambda_1, \ldots, \lambda_k\}$.
Let $o_i = (t_{i,1}, \ldots, t_{i,n_i})$ denote observed sequence of cumulative failure times due to mode i, where $0 < t_{i,1} < \ldots < t_{i,n_i} \leq T$.
Let $L(o_i|x_i)$= likelihood for o_i given x_i:

$$L(o_i \mid x_i) = \left[\prod_{l=1}^{n_i} x_i \left\{ e^{-x_i (t_{i,l} - t_{i,l-1})} \right\} \right] e^{-x_i (T - t_{i,n_i})} = x_i^{n_i} e^{-x_i T} \ where \ \ t_{i,0} = 0.$$

Thus,

$$g(x_i \mid o_i) = \frac{\{L(o_i \mid x_i)\} u(x_i)}{\sum_{j=1}^{k} L(o_i \mid \lambda_j) u(\lambda_j)} = \frac{x_i^{n_i} e^{-x_i T}}{\sum_{j=1}^{k} \lambda_j^{n_i} e^{-\lambda_j T}} \ for \ x_i \in \{\lambda_1, \ldots, \lambda_k\}$$

and

$$E(X_i \mid o_i) = \sum_{l=1}^{k} \lambda_l \left\{ \frac{\lambda_l^{n_i} e^{-\lambda_l T}}{\sum_{j=1}^{k} \lambda_j^{n_i} e^{-\lambda_j T}} \right\} = \frac{\sum_{l=1}^{k} \lambda_l^{n_i + 1} e^{-\lambda_l T}}{\sum_{j=1}^{k} \lambda_j^{n_i} e^{-\lambda_j T}}$$

Posterior Mean Continued.

Consider unobserved failure mode i∈unobs with unknown failure rate $x_i \in \{\lambda_1, \ldots, \lambda_k\}$. Let o_i denote the observation that $n_i = 0$. Let $L(o_i|x_i)$= likelihood for o_i given x_i. Then,

$$L(o_i \mid x_i) = e^{-x_i T}$$

$$g(x_i \mid o_i) = \frac{\{L(o_i \mid x_i)\} u(x_i)}{\sum_{j=1}^{k} L(o_i \mid \lambda_j) u(\lambda_j)} = \frac{e^{-x_i T}}{\sum_{j=1}^{k} e^{-\lambda_j T}} \quad for\ x_i \in \{\lambda_1, \ldots, \lambda_k\}$$

$$E(X_i \mid o_i) = \sum_{l=1}^{k} \lambda_l \left\{ \frac{e^{-\lambda_l T}}{\sum_{j=1}^{k} e^{-\lambda_j T}} \right\} = \frac{\sum_{l=1}^{k} \lambda_l e^{-\lambda_l T}}{\sum_{j=1}^{k} e^{-\lambda_j T}}$$

From I. and II., since $n_i = 0$ for i∈unobs,

$$E(X_i \mid o_i) = \frac{\sum_{l=1}^{k} \lambda_l^{n_i+1} e^{-\lambda_l T}}{\sum_{j=1}^{k} \lambda_j^{n_i} e^{-\lambda_j T}} \text{ for } i = 1, \ldots, k,$$

Interpretation of Posterior Mean.

Let M(t) = number of distinct failure modes surfaced during test by t.

$$M(t) = \sum_{i=1}^{k} I_i(t) \text{ where } I_i(t) = \begin{cases} 1 & \text{if mode } i \text{ occurs by } T \\ 0 & \text{otherwise} \end{cases}$$

$$\mu(t) = E[M(t)] = \sum_{i=1}^{k} E[I_i(t)] = k - \sum_{j=1}^{k} e^{-\lambda_j t}. \text{ Thus } \sum_{j=1}^{k} e^{-\lambda_j t} = k - \mu(t).$$

Let

$$h(t) = \frac{d\mu(t)}{dt}. \text{ Thus } h(t) = \sum_{j=1}^{k} \lambda_j e^{-\lambda_j t}$$

$$h^{(1)}(t) = (-1)\sum_{j=1}^{k} \lambda_j^2 e^{-\lambda_j t}$$

$$h^{(2)}(t) = (-1)^2\sum_{j=1}^{k} \lambda_j^3 e^{-\lambda_j t}$$

$$\vdots$$

$$h^{(n_i-1)}(t) = (-1)^{n_i-1}\sum_{j=1}^{k} \lambda_j^{n_i} e^{-\lambda_j t}$$

Interpretation of Posterior Mean Continued.

For i∈obs, this yields,

$$E(X_i \mid o_i) = \frac{\sum_{l=1}^{k} \lambda_l^{n_i+1} e^{-\lambda_l T}}{\sum_{l=1}^{k} \lambda_l^{n_i} e^{-\lambda_l T}} = \frac{\left\{ \frac{h^{(n_i)}(T)}{(-1)^{n_i}} \right\}}{\left\{ \frac{h^{(n_i-1)}(T)}{(-1)^{n_i-1}} \right\}} = -\frac{h^{(n_i)}(T)}{h^{(n_i-1)}(T)}.$$

$$For\, i \in unobs,\ E(X_i \mid o_i) = \frac{\sum_{l=1}^{k} \lambda_l e^{-\lambda_l T}}{\sum_{l=1}^{k} e^{-\lambda_l T}} = \frac{h(T)}{k-\mu(T)}$$

Let $\rho^*(T)$ denote the assessed projected system failure rate based on the mode failure rate assessments $x_i^* = E(X_i|o_i)$ for i=1,…,k, Then,

$$\rho^*(T) = \sum_{i \in obs} (1-d_i^*) x_i^* + \sum_{i \in unobs} x_i^* = \sum_{i \in obs} (1-d_i^*) \left\{ -\frac{h^{(n_i)}(T)}{h^{(n_i-1)}(T)} \right\} + (k-m)\left(\frac{h(T)}{k-\mu(T)} \right)$$

where m = number of modes surfaced by T.

Parsimonious Model for Expected Number of Modes by t.

Recall expected number of modes by t given $\underline{\lambda} = (\lambda_1,\ldots,\lambda_k)$ is

$$\mu(t;\underline{\lambda}) = k - \sum_{i=1}^{k} e^{-\lambda_i T}$$

We shall assume $\lambda_1,\ldots,\lambda_k$ is a realization of a random sample from a gamma distribution with density

$$f(x) = \begin{cases} \dfrac{x^{\alpha} e^{-x/\beta}}{\Gamma(\alpha+1)\beta^{\alpha+1}} & for\ x > 0; \\ 0 & otherwise \end{cases}$$

Let $\Lambda_1,\ldots,\Lambda_k$ be independent identically distributed gamma random variables with density f(x). Consider $\mu(t; \underline{\Lambda})$ where $\underline{\Lambda}=(\Lambda_1,\ldots,\Lambda_k)$. We shall approximate $\mu(t;\underline{\lambda})$ by $\mu_k(t;\alpha,\beta)=E[\mu(t;\underline{\Lambda})]$, the expected value of $\mu(t;\underline{\Lambda})$ w.r.t. $\underline{\Lambda}$. Can show (AMSAA Growth Guide)

$$\mu_k(t;\alpha,\beta) = k\left\{1 - (1+\beta t)^{-(\alpha+1)}\right\}$$

Model Approximation for Posterior Mean.

We shall use $\mu_k(t;\alpha,\beta)$ and its derivatives to approximate $x_i^* = E(X_i|o_i)$ for i=1,…,k. Let $\lambda_k=E[\Lambda_1+\ldots+\Lambda_k]=k\beta(\alpha+1)$.

Define
$$h_k(t;\alpha,\beta)=\frac{d\mu_k(t;\alpha,\beta)}{dt}=\frac{\lambda_k}{(1+\beta t)^{\alpha+2}}.$$

Note,
$$h_k^{(1)}(t;\alpha,\beta)=\frac{(-1)(\alpha+2)\lambda_k\beta}{(1+\beta t)^{\alpha+3}}$$

$$h_k^{(2)}(t;\alpha,\beta)=\frac{(-1)^2(\alpha+2)(\alpha+3)\lambda_k\beta^2}{(1+\beta t)^{\alpha+4}}$$

$$\vdots$$

$$h_k^{(n_i-1)}(t;\alpha,\beta)=\frac{(-1)^{n_i-1}(\alpha+2)\ldots(\alpha+n_i)\lambda_k\beta^{n_i-1}}{(1+\beta t)^{\alpha+n_i+1}}$$

Let $x^*_{i,a,k}$ denote the approximation of $x_i^* = E(X_i|o_i)$ based on using $\mu_k(t;\alpha,\beta)=E[\mu(t;\underline{\Lambda})]$ to approximate $\mu(t;\underline{\lambda})$. Thus

$$x^*_{i,a,k}=\begin{cases}-\dfrac{h_k^{(n_i)}(T;\alpha,\beta)}{h_k^{(n_i-1)}(T;\alpha,\beta)} & for\ i\in obs;\\[2ex] \dfrac{h_k(T;\alpha,\beta)}{k-\mu_k(T;\alpha,\beta)} & for\ i\in unobs\end{cases}$$

Model Approximation for Posterior Mean Continued.

For i∈obs, $x^*_{i,a,k} = \dfrac{(\alpha + n_i + 1)\beta}{1 + \beta T}$

For i∈unobs $x^*_{i,a,k} = \dfrac{\left\{\dfrac{\lambda_k}{(1+\beta T)^{\alpha+2}}\right\}}{k - k\left\{1 - (1+\beta T)^{-(\alpha+1)}\right\}}$

$$= \frac{k\beta(\alpha+1)}{(1+\beta T)^{\alpha+2}\left\{k(1+\beta T)^{-(\alpha+1)}\right\}}$$

$$= \frac{\beta(\alpha+1)}{1+\beta T}$$

Reliability Projection using the Parsimonious Model Approximation.

Let $\rho^*_{a,k}(T)$ denote the failure rate projection based on the d^*_i for $i \in obs$ and the $x^*_{i,a,k}$ for $i=1,\ldots,k$. Thus,

$$\rho^*_{a,k}(T) = \sum_{i \in obs} (1-d_i^*) x^*_{i,a,k} + \sum_{i \in unobs} x^*_{i,a,k}$$

This yields,

$$\rho^*_{a,k}(T) = \sum_{i \in obs} (1-d_i^*) \left\{ \frac{(\alpha + n_i + 1)\beta}{1+\beta T} \right\} + (k-m) \left\{ \frac{\beta(\alpha+1)}{1+\beta T} \right\}$$

$$= \sum_{i \in obs} (1-d_i^*) \left\{ \frac{(\alpha + n_i + 1)\beta}{1+\beta T} \right\} + (1-\frac{m}{k}) \left\{ \frac{\lambda_k}{1+\beta T} \right\}$$

Reliability Projection based on the MLEs for Gamma Parameters.

Shall use MLE's for gamma parameters given data m and $\underline{n}=(n_1,\ldots,n_k)$. Let N_i denote the random variable for the number of failures due to mode i that occurs during [0,T]. Let $w(s_i;\alpha,\beta)$ denote the marginal density for the compound random variable N_i. Then [Martz & Waller]

$$w(s_i;\alpha,\beta)=\frac{T^{s_i}\Gamma(s_i+\alpha+1)}{\left\{s_i!\,\beta^{\alpha+1}\Gamma(\alpha+1)\right\}\left(T+\frac{1}{\beta}\right)^{s_i+\alpha+1}}\ for\ s_i=0,1,2,\ldots$$

Reliability Projection based on the MLEs for Gamma Parameters Continued.

Likelihood for (α,β) given m and $\underline{n}$ is

$$L(\alpha,\beta;m,\underline{n}) = \prod_{i=1}^{k} w(n_i;\alpha,\beta). \quad Note\ n_i = 0\ for\ i \in unobs$$

Assuming k potential failure modes, let $\hat{\alpha}_k, \hat{\beta}_k$ denote the MLEs for α, β, respectively. Also let $\hat{\lambda}_k = k\hat{\beta}_k(\hat{\alpha}_k + 1)$. Can show (Martz & Waller, Chapter 7)

$$\hat{\lambda}_k = \frac{n}{T}\ where\ n = \sum_{i=1}^{k} n_i \text{ and } \hat{\beta}_k = \frac{y_k}{T}\ where \left(\frac{n}{y_k}\right)\ln(1+y_k) - \sum_{j\in obs}\sum_{i=1}^{n_j-1}\frac{1}{1+\left(iky_k/n\right)} = m$$

The inner sum is defined to be zero when n_j=1.

From the above equations can find $(\hat{\lambda}_\infty, \hat{\beta}_\infty, \hat{\alpha}_\infty) = \lim_{k\to\infty}(\hat{\lambda}_k, \hat{\beta}_k, \hat{\alpha}_k)$

Note $\hat{\lambda}_\infty = \frac{n}{T}$ and $\hat{\beta}_\infty = \frac{y_\infty}{T}$ where y_∞ is the unique positive solution y that satisfies

$\left(\frac{n}{y}\right)\ln(1+y) = m$. It follows that $\hat{\alpha}_\infty = -1$.

Reliability Projection based on MLEs for Gamma Parameters Continued.

For i=1,…,k let $\hat{x}_{i,k}$ denote the statistical estimate of $x^*_{i,a,k}$ based on the MLEs $\hat{\alpha}_k, \hat{\beta}_k$ for α, β, respectively. Let $\hat{\rho}_k(T)$ denote the projected failure rate assessment obtained from $\rho^*_{a,k}(T)$ by replacing α, β and λ_k by $\hat{\alpha}_k, \hat{\beta}_k$ and $\hat{\lambda}_k = k\hat{\beta}_k(\hat{\alpha}_k + 1)$, respectively. Thus

$$\hat{\rho}_k(T) = \sum_{i \in obs} (1 - d_i^*)\hat{x}_{i,k} + \sum_{i \in unobs} \hat{x}_{i,k}$$

By definition $\hat{x}_{i,k} = \dfrac{(\hat{\alpha}_k + n_i + 1)\hat{\beta}_k}{1 + \hat{\beta}_k T} \; for\ i = 1,\ldots,k$.

Therefore,

$$\hat{\rho}_k(T) = \sum_{i \in obs} (1 - d_i^*)\left\{\frac{(\hat{\alpha}_k + n_i + 1)\hat{\beta}_k}{1 + \hat{\beta}_k T}\right\} + \sum_{i \in unobs}\left\{\frac{(\hat{\alpha}_k + 1)\hat{\beta}_k}{1 + \hat{\beta}_k T}\right\}$$

$$= \sum_{i \in obs} (1 - d_i^*)\left(\frac{\hat{\beta}_k T}{1 + \hat{\beta}_k T}\right)\left(\frac{(\hat{\alpha}_k + n_i + 1)}{T}\right) + (k - m)\left\{\frac{\hat{\beta}_k(\hat{\alpha}_k + 1)}{1 + \hat{\beta}_k T}\right\}$$

$$= \sum_{i \in obs} (1 - d_i^*)\left(\frac{\hat{\beta}_k T}{1 + \hat{\beta}_k T}\right)\left(\frac{(n_i + \hat{\alpha}_k + 1)}{T}\right) + \left(1 - m/k\right)\left(\frac{n/T}{1 + \hat{\beta}_k T}\right)$$

since $k\hat{\beta}_k(\hat{\alpha}_k + 1) = n/T$

Reliability Projection based on MLEs for Gamma Parameters Continued.

Note $\hat{\alpha}_k + 1 = \dfrac{n}{k\hat{\beta}_k T}$. Therefore,

$$\hat{\rho}_k(T) = \sum_{i \in obs} (1 - d_i^*) \left(\frac{\hat{\beta}_k T}{1 + \hat{\beta}_k T} \right) \left(\frac{n_i}{T} + \frac{n}{k\hat{\beta}_k T^2} \right) + \left(1 - m/k\right) \left(\frac{n/T}{1 + \hat{\beta}_k T} \right)$$

This yields,

$$\hat{\rho}_\infty(T) = \lim_{k \to \infty} \hat{\rho}_k(T) = \sum_{i \in obs} (1 - d_i^*) \left(\frac{\hat{\beta}_\infty T}{1 + \hat{\beta}_\infty T} \right) \left(\frac{n_i}{T} \right) + \left(\frac{n/T}{1 + \hat{\beta}_\infty T} \right)$$

Reliability Projection based on MMEs for Gamma Parameters

Let $\overline{\Lambda}$ and M^2 denote the random variables that take on the values $\overline{\lambda}$ and m^2 respectively where

$$\overline{\lambda} = \frac{1}{k}\sum_{i=1}^{k}\hat{\lambda}_i \text{ and } m^2 = \frac{1}{k}\sum_{i=1}^{k}\hat{\lambda}_i^2 \text{ with } \hat{\lambda}_i = \frac{n_i}{T}$$

One can show $E[\overline{\Lambda};\alpha,\beta] = \beta(\alpha+1)$ and $E[M^2;\alpha,\beta] = \dfrac{\beta^2(\alpha+1)\left[T(2+\alpha)+ {}^{1}\!/\!_{\beta}\right]}{T}$

In [Martz and Waller], the MMEs for α and β, denoted by $\tilde{\alpha}_k, \tilde{\beta}_k$ respectively, are implicitly defined as follows:

$$\overline{\lambda} = \tilde{\beta}_k(\tilde{\alpha}_k+1) \text{ and } m^2 = \frac{\tilde{\beta}_k^2(\tilde{\alpha}_k+1)\left[T(2+\tilde{\alpha}_k)+ {}^{1}\!/\!_{\tilde{\beta}_k}\right]}{T}$$

From these equations it follows

$$\tilde{\lambda}_k = k\tilde{\beta}_k(\tilde{\alpha}_k+1) = \frac{n}{T} \text{ and } \tilde{\beta}_k = \frac{\sum_{j\in obs} n_j^2 - \frac{n^2}{k} - n}{T\cdot n}$$

Reliability Projection based on MMEs for Gamma Parameters Continued

Let $(\tilde{\lambda}_\infty, \tilde{\beta}_\infty) = \lim_{k\to\infty}(\tilde{\lambda}_k, \tilde{\beta}_k)$. One obtains

$$\tilde{\lambda}_\infty = \frac{n}{T} \text{ and } \tilde{\beta}_\infty = \frac{1}{T}\left(\frac{\sum_{j\in obs} n_j^2}{n} - 1\right)$$

Let $\tilde{\rho}_k(T)$ denote the projection for the mitigated system failure rate based on the finite k MMEs. Then

$$\tilde{\rho}_k(T) = \sum_{i\in obs}(1-d_i^*)\left(\frac{\tilde{\beta}_k T}{1+\tilde{\beta}_k T}\right)\left(\frac{n_i}{T} + \frac{n}{k\tilde{\beta}_k T^2}\right) + \left(1 - \frac{m}{k}\right)\left(\frac{n/T}{1+\tilde{\beta}_k T}\right)$$

$$\tilde{\rho}_\infty(T) = \lim_{k\to\infty}\tilde{\rho}_k(T) = \sum_{i\in obs}(1-d_i^*)\left(\frac{\tilde{\beta}_\infty T}{1+\tilde{\beta}_\infty T}\right)\left(\frac{n_i}{T}\right) + \left(\frac{n/T}{1+\tilde{\beta}_\infty T}\right)$$

Noninformative Prior Bayesian Approach with A and B-Mode Classification

- *A-mode: no corrective action planned even if surfaced.*
- *B-mode: if surfaced, will be mitigated.*

Approach still applies for two failure mode categories. Apply previous procedure to set of B-modes to obtain projection of system failure rate due to the B-modes, say $\hat{\rho}_k(T)$.

The prior now pertains to $x_i \in \{\lambda_1, \ldots \lambda_k\}$, the initial failure rates of the k B-modes. Likewise, the data m and N_i pertain to the B-modes.

Then the projection for the system mitigated failure rate using MLEs is

$$\hat{\rho}_{A+B,k}(T) = \frac{N_A}{T} + \hat{\rho}_k(T)$$

and

$$\hat{\rho}_{A+B,\infty}(T) = \lim_{k \to \infty} \hat{\rho}_{A+B,k}(T) = \frac{N_A}{T} + \hat{\rho}_\infty(T)$$

where N_A denotes the number of A-mode failures in test.

The same comments apply to obtaining projections for two classifications using the MMEs for α and β.

Simulation Overview

The simulation consists of the following steps:

1. Specify inputs.
2. Generate failure rates.
3. Calculate mode failure times.
4. Calculate first occurrence times and number of failures during test for each mode.
5. Generate FEFs from beta distribution with mean = 0.80 and coefficient of variation = 0.10.
6. Calculate MTBF projections.
7. Reclassify repeat A-modes.
8. Recalculate MTBF projections.

	Simulated	Surfaced	Distribution
A-Modes	50	23	Gamma
B-Modes	100	45	Gamma

Table 1. Simulated/Surfaced Modes.

	A-Modes	B-Modes
Shape - α	3.3333	3.3333
Scale - β	0.0002	0.0002

Table 2. Gamma Parameters.

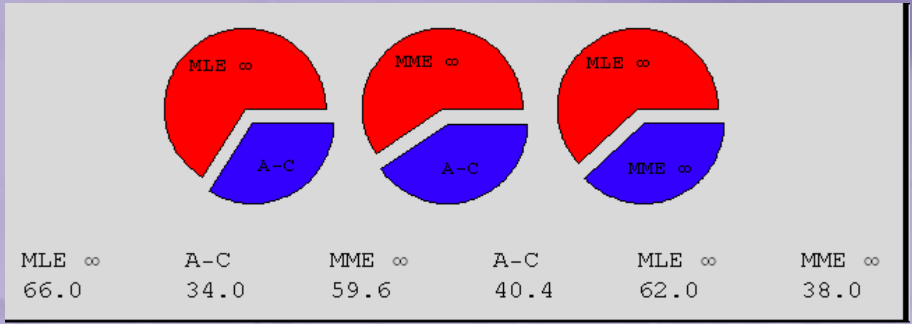
•Results obtained from simulating 1,000 tests of length 1,000 hours.

•Mode failure rates and FEFs regenerated for each test.

Simulation Results (Gamma)

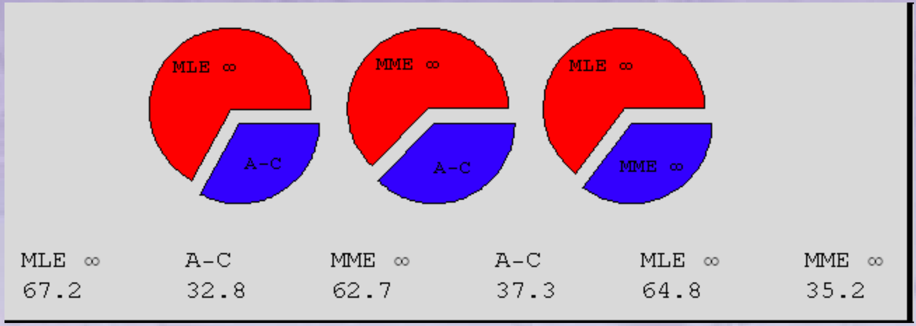
	Actual	MLE ∞	MME ∞	A-C
MTBF	14.15	13.94	13.42	13.26

Table 3. Two Categories.



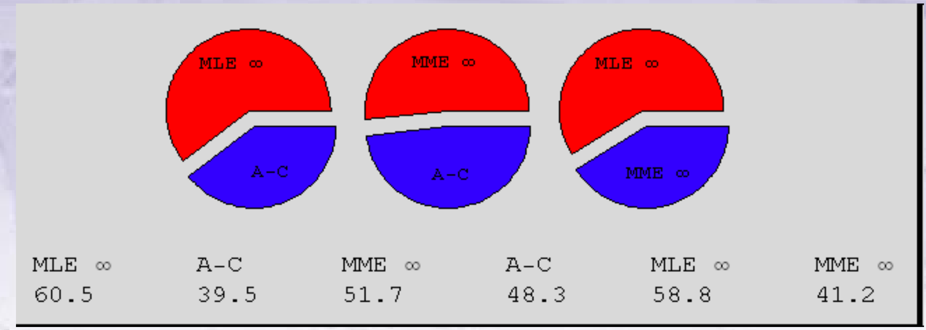
	Actual	MLE ∞	MME ∞	A-C
MTBF	14.15	13.92	13.41	13.22

Table 4. One Category.



	Actual	MLE ∞	MME ∞	A-C
MTBF	15.43	15.54	14.73	15.09

Table 5. Two Categories, after Reclassification.

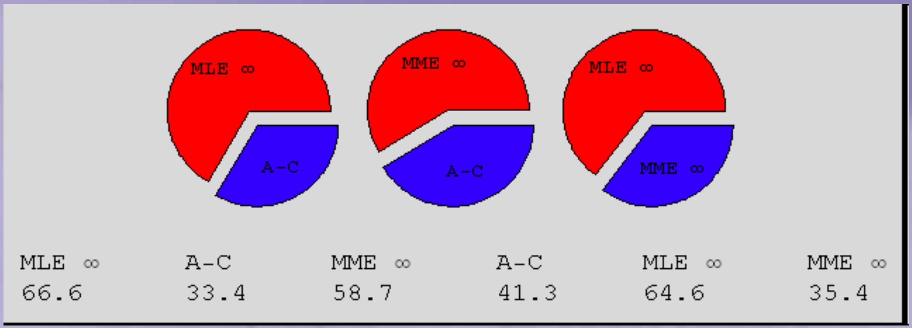


8 modes reclassified on average.

Simulation Results (Weibull)

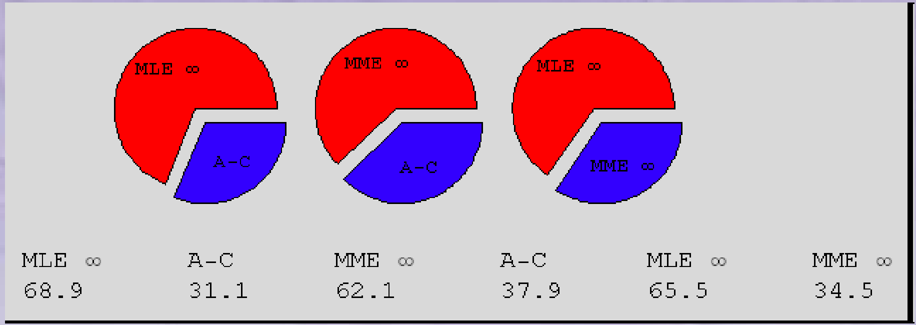
	Actual	MLE ∞	MME ∞	A-C
MTBF	14.31	13.99	13.45	13.29

Table 6. Two Categories.



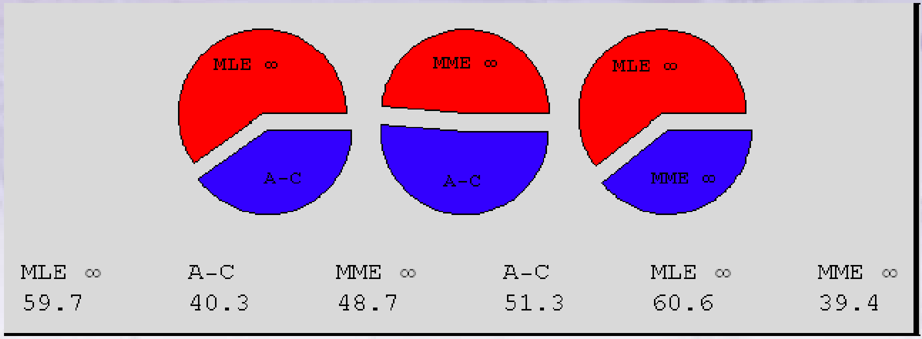
	Actual	MLE ∞	MME ∞	A-C
MTBF	14.31	13.98	13.46	13.27

Table 7. One Category.



	Actual	MLE ∞	MME ∞	A-C
MTBF	15.67	15.65	14.82	15.20

Table 8. Two Categories, after Reclassification.



8 modes reclassified on average.

Cost versus Reliability Tradeoff Analysis

Let $Z \subset obs$ be a candidate set of observed modes to receive fixes following the test phase. Based on a study of the underlying root causes of failure, fixes could be devised with associated FEF assessments d_i^* for $i \in obs$. The corresponding projection for the resulting system failure rate would be for large k (using MLEs for α and β).

$$\hat{\rho}_\infty(T;Z) = \sum_{i \in Z}(1 - d_i^*)\hat{x}_{i,\infty} + \sum_{i \in obs - Z}\hat{x}_{i,\infty} + \sum_{i \in unobs}\hat{x}_{i,\infty}$$

One could also assess the cost, $c^*(Z)$, of implementing all the fixes for modes $i \in Z$.

A plot of the projected MTBF vs. associated cost for a number of selected $Z \subset obs$ would be useful in identifying a least cost solution Z to meet a reliability goal. In place of using MLEs, one could use MME based assessments.

Projection methods whose estimation procedures for a given data set treat A-mode and B-mode data differently beyond differentiation with regard to FEFs are not suitable for performing cost/reliability tradeoff analysis. Such methods include those that utilize an estimate of the expected B-mode failure rate due to the unsurfaced B-modes.

Extensions of Approach to Situations where Fixes Need Not be Delayed

I. Using the n_i for estimation.

- Unknown mode failure rate $x_i \in \{\lambda_1,\ldots,\lambda_k\}$ either generates observed data $n_i=0$ or $o_i = (t_{i,1},\ldots,t_{i,n_{i,1}}, v_i, t_{i,n_{i,1}+1},\ldots,t_{i,n_{i,2}})$ where $0 < t_{i,1} < \ldots < t_{i,n_{i,1}} \le v_i < t_{i,n_{i,1}+1} < t_{i,n_{i,2}} \le T$

  For the above data, $n_{i,1} \ge 1$ and $n_{i,1} + n_{i,2} = n_i$. The $t_{i,j}$ are the cumulative failure times for mode i and v_i denotes the time at which the fix to mode i is implemented.
- Can show

$$E(X_i \mid o_i) = -\frac{h^{(n_i)}(v_i + (1-d_i)(T-v_i))}{h^{(n_i-1)}(v_i + (1-d_i)(T-v_i))}$$

where d_i denotes realized FEF for mode i.

- The assessment of $E(X_i|o_i)$ depends on v_j and d_j^* for all $j \in obs$.

II. Using failure mode first occurrence times.

- Unknown mode failure rate x_i either generates observed data $n_i=0$ or $o_i=t_{i,1}$ where $t_{i,1}$ is the first occurrence time for mode i.
- Can show

$$E(X_i \mid o_i) = -\frac{h^{(1)}(t_{i,1})}{h(t_{i,1})}$$

- The assessment of $E(X_i|o_i)$ will not depend on any of the v_j or d_j^*.

Concluding Remarks

Noninformative Prior Bayesian Approach useful in deriving reliability growth projection methods:

for case where all fixes delayed,

for situation where not all fixes need be delayed,

potentially for deriving discrete projection methodology.

For current simulations, described procedures compare favorably to the standard adopted by the International Electrotechnical Commission (AMSAA-Crow Projection Model).

Method does not require one to distinguish for estimation purposes between A-modes and B-modes other than through FEFs.

Can also be used for case when failure modes can be split into inherent A-modes and B-modes.

Method suitable for cost versus reliability tradeoff analysis for modes that are not inherently A-modes.

Model and estimation procedures only require reference to FEFs for surfaced modes.

Comparable simulation results obtained when failure rates drawn from Weibull or lognormal distributions with same mean and variance as the gamma.

References

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